7.2 Noise and source-count slope

This is easily investigated by simulation; generate samples from a power-law distribution by the methods of ¶6.5, add Gaussian noise and use the maximum-likelihood estimator of ¶6.1 to get the index. A key assumption is that the noise is *additive*. The lower limit of the power law is of interest. If we have a lower limit to the distribution of S_0, a flux limit for our "survey" of S_{lim} and a noise level σ, then for a realistic simulation we must have

$$S_0 << S_{\text{lim}} \leq \sigma.$$

Since most of the "sources" are at small fluxes, we find (as is common in these simulations) that we generate many sources, only to throw them away because even with a noise boost they do not come above the flux limit.
We used $S_0 = 0.1$, and $S_{\text{lim}} = \sigma = 1$, for an integral source count of index 2. The results are in the figures.
This problem can in principle be solved analytically, as it is a convolution of the source count with the noise distribution. The lack of suitable Fourier transforms (unless you are happy with some black belt integrations) is an obstacle to this approach.

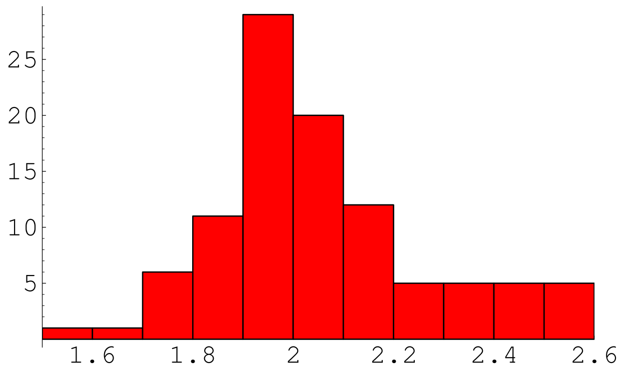

Figure 1: Distribution of estimates of the power-law index, for 100 repetitions on a set of 10000 sources. In this case there is no noise: $\sigma = 0$.

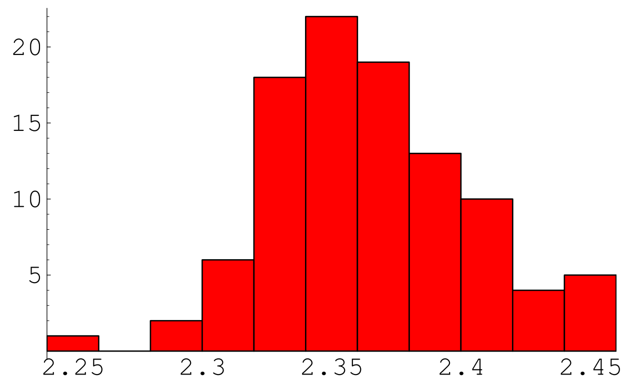

Figure 2: Distribution of estimates of the power-law index, for 100 repetitions on a set of 10000 sources. In this case there is noise: $\sigma = 1$. The measured source count has steepened.